1. **Data and functional parallelism** [5 pts.]

Identify *all* sources of data and functional parallelism in the following program segment:

```plaintext
f1: for i ← 0 to 99 do
s1:   a[i] ← b[i] + c[i]
 s2:   d[i] ← e[i] + f[i]
 s3:   g[i] ← a[i] + d[i]
       end for
s4:   h[0] = 10^10;

f2: for i ← 1 to 99 do
s5:   h[i] ← sqrt(h[i-1])
       end for

f3: for i ← 0 to 99 do
s6:   m[i] ← h[i] + a[30]
       end for
```
2. **Static versus self-scheduling** [5 pts.]

Suppose we are going to speed up the execution of the data clustering algorithm presented in Chapter 1 of the textbook by using \( p \) processors to generate the \( D \)-dimensional vectors for each of the \( N \) documents. One approach would be to preallocate about \( N/p \) documents to each processor. Another approach would be to put the documents on a list and let processors remove documents as fast as they could process them. Discuss one advantage of each approach.
3. **MPI reductions** [5 pts.]

Write a parallel program that computes the sum $1+2+\ldots+p$ in the following manner: Each process $i$ assigned the value $i+1$ to an integer variable, and then the processes perform a sum reduction of these values. Process 0 should print the result of the reduction. Assume a total of $p$ processes.
4. Point-to-point communication [5 pts.]

Consider the following code segment:

```c
int x, y, z, i, j, k;
int id, p;
MPI_Status status;
MPI_Comm_rank(MPI_COMM_WORLD, &id);
MPI_Comm_size(MPI_COMM_WORLD, &p);
...
y = id;

j = id - 1;
if (j < 0) j = p - 1;
k = id + 1;
if (k > p - 1) k = 0;
MPI_Recv(&x, 1, MPI_FLOAT, k, 0, MPI_COMM_WORLD, &status);
MPI_Send(&y, 1, MPI_FLOAT, j, 0, MPI_COMM_WORLD);
z = x + y;
```

Fix the MPI-related bug in this program and state what is the value of $z$ at the end of the program.
5. **Performance analysis [5 pts.]**

The execution time of Floyd’s algorithm is computed in the textbook as:

\[
n^2\left[\frac{n}{p}\right] \chi + n (\log p) \lambda + \lceil \log p \rceil 4^n \frac{n}{\beta}
\]

Here, we assume that \(n=1000\), \(\chi = 25.5\) nsec, \(\lambda = 250\) µsec, and \(\beta = 10^7\).

The value of this formula for \(p=1, 2, 3, 4, 5, 6, 7,\) and \(8\) is respectively:

25.5000, 13.0000, 9.0170, 6.8750, 5.8500, 5.0085, 4.3965, and 3.9375

Compute the Karp-Flatt serial fraction for \(p=2, 4,\) and \(8\). Explain the meaning of the values obtained for the serial fraction. Hint: Watch the units!
<Extra credit>. Ceilings and floors [2 pts.]

Prove the following formula:

$$\left\lfloor \frac{m + n - 1}{n} \right\rfloor = \left\lfloor \frac{m}{n} \right\rfloor$$